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RHEOLOGICAL EQUATIONS OF STATE OF WEAKLY CONCENTRATED SUSPENSIONS OF DEFORMABLE ELLIPSOIDAL PARTICLES

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In [1], from the standpoint of a structural-continuous approach [2, 3], rheological equations of state are obtained for dilute suspensions of deformable ellipsoidal particles, having internal elasticity and viscosity, with a dispersion medium which is a Newtonian liquid. In the present article these results are generalized for larger concentrations. Taking account of the effect of the hydrodynamic interaction of suspended particles on the rheological behavior of a suspension is effected using the Simha method [4].

As in [1], we shall model the suspended particles by an ellipsoid, having an internal linear elasticity and a linear viscosity (a Voigt body), changing its dimensions during the process of interaction with the dispersion medium, but conserving its volume and retaining the form of an ellipsoid of revolution. To set up the rheological equations of state of the suspensions under consideration using a structural-continuous approach, it is necessary to determine the perturbations introduced into the inhomogeneous flow of a dispersion medium by a suspended particle; here, to take account of the hydrodynamic interaction of the suspended particles, the boundary conditions "at infinity," in accordance with [4], must be referred to the surface of a sphere screening the particle; the sphere has a center coinciding with the center of the particle, and a radius $R = (ab^2/\Phi)^{1/3}$, where $2a$ and b are the length of the axis of rotation and the equatorial radius of the particle, respectively; Φ is the volumetric concentration of suspended particles.

We shall seek the solution of the hydrodynamic problem in the Stokes approximation by the method of successive approximations [5]. As a first approximation we take the solution obtained in [1] for the case where an unbounded dispersion flows around the particle, but where the boundary conditions "at infinity" are referred to the surface of a sphere, whose radius considerably exceeds the effective radius of the particle. In a movable system of coordinates x_i, with its origin at the center of the particle and axes coinciding in direction with the directions of the axes of an ellipsoidal particle, this solution has the form

$$u_i = u_{0i} + \frac{\partial}{\partial x_i}(D_j\chi_j) - \varepsilon_{ijk}K_j\frac{\partial \chi_j}{\partial x_k} + c_{jk}x_j\frac{\partial^2\Omega}{\partial x_i \partial x_k} - c_{ij}\frac{\partial\Omega}{\partial x_j} - \frac{4}{3R^3}(c_{ki} - c_{ik})x_k + \frac{4x_i\psi}{R^5} + \frac{5(R^2 - r^2)}{R^5}\frac{\partial\psi}{\partial x_i},$$
$$p = p_0 + 2\mu c_{ij}\frac{\partial^2\Omega}{\partial x_i\partial x_j}, \qquad (1)$$

where u_i is the velocity; p is the pressure; u_{0i}, p_0 are the velocity and the pressure of the unperturbed flow; r is the modulus of the radius-vector; μ is the dynamic viscosity coefficient of the dispersion medium; Ω, χ_j, D_j, K_j are values determined in [6]; c_{ij} are values determined in [1]; $\psi = c_{ij}x_ix_j$; ϵ_{ijk} is a skew-symmetric Kronecker symbol.

The first approximation (1) does not satisfy the boundary conditions at the surface of the particle; here, the divergences do not exceed values of the order of $O(R^{-3})$.

We obtain the second approximation of the problem under consideration, adding to (1) a partial solution of the problem, satisfying the following boundary conditions:

$$u_i|_w = \frac{4}{3R^3}(c_{ki} - c_{ik})x_k - \frac{5}{R^5}\frac{\partial\psi}{\partial x_i},$$
$$u_i \to 0 \quad \text{for} \quad r \to \infty,$$

where $u_i|_w$ is the velocity at the surface of the particle. This partial solution has the form

$$u_i = \frac{\partial}{\partial x_i}(Q_j\chi_j) - \varepsilon_{ijk}H_j\frac{\partial\chi_j}{\partial x_k} + B_{jk}x_j\frac{\partial^2\Omega}{\partial x_i\partial x_k} - B_{ij}\frac{\partial\Omega}{\partial x_j}, \quad p = 2\mu B_{ij}\frac{\partial^2\Omega}{\partial x_i\partial x_j}, \qquad (2)$$

where

$$Q_1 = -\frac{3d_{32}}{4R^3b^2\alpha_0'\beta_0'};$$

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$$Q_2 = \frac{5}{2R^3B^2\beta_0'}\left[-(\alpha_0+\beta_0)\,d_{31} + (b^2-a^2)(\omega_{31}+\omega_2)\,\beta_0'\right];$$

$$Q_3 = -\frac{5}{2R^3B^2\beta_0'}\left[(\alpha_0+\beta_0)\,d_{21} + (a^2-b^2)(\omega_{12}+\omega_3)\,\beta_0'\right];$$

$$H_1 = 2(B_{22}-B_{33})\,b^2; \quad H_2 = 2(b^2B_{33}-a^2B_{11});$$

$$H_3 = 2(a^2B_{11}-b^2B_{22});$$

$$B_{11} = \frac{5d_{11}}{18\beta_0''^2R^3} + \frac{5}{12R^3\left[\alpha_0+2\beta_0-2\beta_0'(a^2+b^2)\right]}\,\frac{\dot a}{a\beta_0''};$$

$$B_{22} = \frac{5d_{22}}{8b^4\alpha_0'^2R^3} + \frac{5(\beta_0''-\alpha_0'')(2b^2\alpha_0'+3\beta_0'')}{72b^4\alpha_0'^2\beta_0''^2R^3}\,d_{11} +$$

$$+\frac{50\beta_0'a^2\left[2\beta_0'a^2-(\alpha_0+2\beta_0)\right]}{R^3(\alpha_0+2\beta_0)\left[\alpha_0+2\beta_0-2\beta_0'(a^2+b^2)\right]}\left[\frac{1}{24a^2\beta_0\beta_0''} - \frac{1}{\left[2\beta_0'a^2-(\alpha_0+2\beta_0)\right]a\beta_0''}\right]\dot a;$$

$$B_{33} = \frac{5d_{33}}{8b^4\alpha_0'^2R^3} + \frac{5(\beta_0''-\alpha_0'')(2b^2\alpha_0'+3\beta_0'')}{72b^4\alpha_0'^2\beta_0''^2R^3}\,d_{11} +$$

$$+\frac{50\beta_0'a^2\left[2\beta_0'a^2-(\alpha_0+2\beta_0)\right]}{R^3(\alpha_0+2\beta_0)\left[\alpha_0+2\beta_0-2\beta_0'(a^2+b^2)\right]}\left[\frac{1}{24a^2\beta_0\beta_0''} - \frac{1}{\left[2\beta_0'a^2-(\alpha_0+2\beta_0)\right]a\beta_0''}\right]\dot a;$$

$$B_{12} = \{\left[15\alpha_0(\alpha_0+\beta_0) - 4b^2\beta_0'(\beta_0-\alpha_0)\right]d_{12} + \left[4b^2(a^2+b^2)\beta_0'^2 -\right.$$
$$\left.- 15(a^2-b^2)\,\alpha_0\beta_0'\right](\omega_{12}+\omega_3)\}\,\frac{1}{12\beta_0'^2B^2R^3};$$

$$B_{21} = \{\left[15\beta_0(\alpha_0+\beta_0) + 4a^2\beta_0'(\beta_0-\alpha_0)\right]d_{21} + \left[4a^2(a^2+b^2)\beta_0'^2 +\right.$$
$$\left.+ 15(a^2-b^2)\,\beta_0\beta_0'\right](\omega_{21}-\omega_3)\}\,\frac{1}{12\beta_0'^2B^2R^3};$$

$$B_{13} = \{\left[15\alpha_0(\alpha_0+\beta_0) - 4b^2\beta_0'(\beta_0-\alpha_0)\right]d_{13} + \left[4b^2(a^2+b^2)\beta_0'^2 -\right.$$
$$\left.- 15(a^2-b^2)\,\alpha_0\beta_0'\right](\omega_{13}-\omega_2)\}\,\frac{1}{12\beta_0'^2B^2R^3};$$

$$B_{31} = \{\left[15\beta_0(\alpha_0+\beta_0) + 4a^2\beta_0'(\beta_0-\alpha_0)\right]d_{31} + \left[4a^2(a^2+b^2)\beta_0'^2 +\right.$$
$$\left.+ 15(a^2-b^2)\,\beta_0\beta_0'\right](\omega_{31}+\omega_2)\}\,\frac{1}{12\beta_0'^2B^2R^3};$$

$$B_{23} = \frac{15\beta_0^2d_{23} + 4b^4\alpha_0'^2(\omega_{23}+\omega_1)}{24b^4\alpha_0'^2\beta_0^2R^3}; \quad B_{32} = \frac{15\beta_0^2d_{32} + 4b^4\alpha_0'^2(\omega_{32}-\omega_1)}{24b^4\alpha_0'^2\beta_0^2R^3};$$

d_{ij}, ω_{ij} are the tensor of the deformation rates and the tensor of the vortices of the velocities of the unperturbed flow; ω_i is the angular velocity of the particle; α_0, β_0, α_0', β_0', α_0'', β_0'' are values determined in [6]; $B = a^2\alpha_0 + b^2\beta_0$.

Summing (1) and (2), we obtain a solution of the problem under consideration, making it possible to determine the characteristics of the suspension with an accuracy up to quantities of the order of $O(\Phi^2)$:

$$u_i = u_{0i} + \frac{\partial}{\partial x_i}\left[(D_j+Q_j)\chi_j\right] - \varepsilon_{ijk}\frac{\partial\chi_j}{\partial x_k}(K_j+H_j) +$$

$$+(c_{jk}+B_{jk})\,x_j\frac{\partial^2\Omega}{\partial x_i\partial x_k} - (c_{ij}+B_{ij})\frac{\partial\Omega}{\partial x_j} - \frac{4}{3R^3}(c_{ki}-c_{ik})\,x_k + \frac{4x_i\psi}{R^3} + \frac{5(R^2-r^2)}{R^5}\frac{\partial\psi}{\partial x_i}; \tag{3}$$

$$p = p_0 + 2\mu(c_{ij}+B_{ij})\frac{\partial^2\Omega}{\partial x_i\partial x_j} - \frac{42\mu}{R^5}\psi. \tag{4}$$

As the tensor of the stresses in the suspension, following Landau [7], we take the tensor, averaged over the volume of the cell of the suspension under consideration, of the stresses arising in the flow of a dispersion medium perturbed by a suspended particle. Using the generalized Newton law for the dispersion medium, the solution of the problem of the perturbations, induced in the flow of a dispersion medium by a suspended particle (3), (4), and going over in the averaging from integration over the volume over of a cell of the suspension to integration over its surface, we obtain

$$\sigma_{ij} = -p_0\delta_{ij} + 2\mu d_{ij} + (8\mu\Phi/ab^2)(c_{ij}+B_{ij}). \tag{5}$$

The tensor of the stresses of the suspension (5) is written in a movable system of coordinates, connected with a particle.

Let us consider acting on a suspended particle from the side of the dispersion medium. Using the generalized Newton law and the solution (3), (4), we obtain the hydrodynamic force

$$p_i = -p_0 g f_i + \frac{8\mu g}{ab^2}(c_{ij}+B_{ij}) f_j - 4g\mu\,[\alpha_0 (c_{11}+B_{11}) + \beta_0 (c_{22}+B_{22}+c_{33}+B_{33})]\, f_i + \frac{10\mu g}{R^3}(c_{ij}+c_{ji}) f_j, \tag{6}$$

where $f_1 = \frac{x_1}{a^2};\quad f_2 = \frac{x_2}{b^2};\quad f_3 = \frac{x_3}{b^2};\quad g = \left(\frac{x_1^2}{a^4} + \frac{x_2^2+x_3^2}{b^4}\right)^{-1/2}$.

As in [1, 5], the principal vector of the hydrodynamic forces acting on a suspended particle is equal to zero.

The orientation of a suspended particle, neglecting its inertia, satisfies the equations

$$M_i + M_i^* = 0, \tag{7}$$

where M_i is the moment of the hydrodynamic forces acting on the particle; M_i^* is the moment of the external forces. The moment of the hydrodynamic forces, as analysis shows, does not differ from the moment obtained in (6) for the case of weakly concentrated solutions of suspensions of rigid ellipsoidal particles. If the external forces acting on the particle are due only to Brownian movement, the principal vector of the forces [9]

$$F_i = -kT\frac{1}{F}\frac{\partial F}{\partial n_i},\quad M_i^* = -kT\varepsilon_{ikm}\frac{n_k}{F}\frac{\partial F}{\partial n_m},$$

where k is the Boltzmann constant; T is the absolute temperature; n_k is the component of the vector oriented along the axis of symmetry of the suspended particle; F is a function of the distribution of the angular positions and the lengths of the axis of symmetry of the suspended particle.

We obtain an equation describing the deformation of a particle, using the principle of virtual displacements with the assumption that the deformation of the particle is homogeneous

$$\frac{\dot a}{a} = -\frac{2ab^2\beta_0'' G\left(\frac{a}{a_0}-\frac{b}{b_0}\right)}{\mu\left(2+3ab^2\frac{\eta}{\mu}\beta_0''\right)}\frac{1}{(1+\Phi M)} + \frac{2+\Phi N}{1+\Phi M}\,\frac{d_{11}}{2+3ab^2\frac{\eta}{\mu}\beta_0''} + \frac{3a\beta_0''\,|F_i|}{4\pi\mu\left(2+3ab^2\beta_0''\frac{\eta}{\mu}\right)}\frac{1}{(1+\Phi M)},$$

where a_0, b_0 are the length of the axis of rotation and the equatorial radius of the particle in an undeformed state:

$$M = \frac{4}{ab^2\left(2+3ab^2\frac{\eta}{\mu}\beta_0''\right)}\left\{\frac{5}{6\left[\alpha_0+2\beta_0-2\beta_0'(a^2+b^2)\right]} - \frac{100\beta_0' a^2(2\beta_0' a^2-\alpha_0-2\beta_0)}{(\alpha_0+2\beta_0)\left[\alpha_0+2\beta_0-2\beta_0'(a^2+b^2)\right]}\left[\frac{1}{24a\beta_0} - \frac{1}{2\beta_0'^2 a^2-(\alpha_0+2\beta_0)}\right]\right\};$$

$$N = \frac{4}{ab^2}\left[\frac{10}{18\beta_0''} + \frac{5\beta_0''}{8b^4\alpha_0'^2} - \frac{10(\beta_0''-\alpha_0'')(2b^2\alpha_0'+3\beta_0'')}{72b^4\alpha_0'^2}\right].$$

To obtain the rheological equations of state of the suspensions under consideration, we use the modulus of an elasticoviscous anisotropic Ericksen liquid, having one internal parameter, i.e., the vector n_i [10, 11], whose direction we connect with the direction of the axis of rotation of the suspended particle, and the modulus, with the length of the semi-axis of rotation, is set n = a:

$$t_{ij} = (c_0 + c_1 d_{km} n_k n_m + c_2 N_k n_k)\delta_{ij} + c_3 n_i n_j + c_4 d_{km} n_k n_m n_i n_j + c_5 N_k n_k n_i n_j + c_6 d_{ij} + c_7 d_{ik} n_k n_j + c_8 d_{jk} n_k n_j + c_9 n_i N_j + c_{10} n_j N_i; \tag{8}$$

$$\dot n_i = \omega_{ij} n_j + \lambda_1 n_i + \lambda_2 d_{km} n_k n_m n_i + \lambda_3 d_{ij} n_j + \lambda_4 \varepsilon_{ijk} M_j^* n_k + \lambda_5 R_j n_j n_i, \tag{9}$$

where t_{ij} is the stress tensor; $N_i = \dot n_i - \omega_{ij} n_j$; c_i, λ_j are rheological functions, depending on $n^2 = n_i n_i$; δ_{ij} is a symmetrical Kronecker symbol.

Considering (8), (9) in a movable system of coordinates, connected with the particle ($n_1 = a$, $n_2 = n_3 = 0$, $\dot n_1 = \dot a$, $n_2 = a\omega_3$, $n_3 = -a\omega_2$), and equating (5) with (8) and (7) with (9), we find the rheological functions c_i, λ_i, entering into (8), (9):

$$c_0 = -p_0,$$

$$c_1 = \frac{2\mu(\beta_0''-\alpha_0'')\Phi}{3a^3b^4\beta_0''\alpha_0''} + \frac{5\mu(\beta_0''-\alpha_0'')(2b^2\alpha_0'+3\beta_0'')}{9a^4b^8\beta_0''^2\alpha_0'^2}\Phi^2,$$

$$c_2 = \frac{2\mu\Phi}{3a^3b^2\beta_0} + \frac{400\mu\beta_0'\left(2a\beta_0' - \alpha_0 - 2\beta_0\right)}{(\alpha_0 + 2\beta_0)\left[\alpha_0 + 2\beta_0 - 2\beta_0'(a^2 + b^2)\right]}\left[\frac{1}{24a^2\beta_0\beta_0''} - \frac{1}{2\beta_0'a^2 - (\alpha_0 + 2\beta_0)a\beta_0''}\right]\Phi^2,$$

$$c_3 = 0; \quad c_4 = \frac{2\mu\Phi}{a^5b^2}\left[\frac{\alpha_0'' + \beta_0''}{b^2\alpha_0'\beta_0''} - \frac{2(\alpha_0 + \beta_0)}{\beta_0'B}\right] + \frac{\mu\Phi^2}{a^4}\left\{-\frac{5}{a^2b^8\alpha_0'^2} + \frac{5}{10\beta_0''^2ab^2} - \frac{5(\beta_0'' - \alpha_0'')(2b^2\alpha_0' + 3\beta_0'')}{9b^8\beta_0''^2\alpha_0'^2} - \right.$$

$$\left. - \frac{2}{b^4}\left[\frac{15(\alpha_0 + \beta_0)^2 + 4(a^2 - b^2)(\beta_0 - \alpha_0)\beta_0'}{3\beta_0'^2B^2}\ \frac{5}{b^4\alpha_0'^2}\right]\right\},$$

$$c_5 = \frac{2\mu\Phi}{a^5b^2}\left[\frac{2(a^2 - b^2)}{B} - \frac{1}{\beta_0''}\right] + \frac{\mu\Phi^2}{a^2}\left\{\frac{-400\beta_0'\left(2\beta_0'a - \alpha_0 - 2\beta_0\right)}{ab^4(\alpha_0 + 2\beta_0)\left[\alpha_0 + 2\beta_0 - 2\beta_0'(a^2 + b^2)\right]} \times \right.$$

$$\left. \times \left[\frac{1}{24a^2\beta_0\beta_0''} - \frac{1}{2\beta_0'a^2 - \alpha_0 - 2\beta_0}\right] - \frac{8(b^4 + a^4)\beta_0' + 15(a^2 + b^2)(\alpha_0 + \beta_0)}{3a^2b^4\beta_0'B^2} + \frac{5}{12a^3b^2\left[\alpha_0 + 2\beta_0 - 2\beta_0'(a^2 + b^2)\right]\beta_0''}\right\},$$

$$c_6 = 2\mu\left(1 + \frac{\Phi}{ab^4\alpha_0'} + \frac{5\Phi^2}{2a^2b^8\alpha_0'^2}\right), \quad c_7 = \frac{4\mu\Phi}{ab^2}\left(\frac{\beta_0}{\beta_0'B} - \frac{1}{2b^2\alpha_0'}\right) + \frac{2\mu\Phi^2}{a^2b^4}\left[\frac{15\beta_0(\alpha_0 + \beta_0) + 4a^2\beta_0'(\beta_0 - \alpha_0)}{3\beta_0'^2B^2} - \frac{5}{2b^4\alpha_0'^2}\right],$$

$$c_8 = \frac{4\mu\Phi}{a^3b^2}\left(\frac{\alpha_0}{\beta_0'B} - \frac{1}{2b^2\alpha_0'}\right) + \frac{2\mu\Phi^2}{a^4b^4}\left[\frac{15\alpha_0(\alpha_0 + \beta_0)}{3\beta_0'^2B^2} - \frac{4b^2\beta_0'(\beta_0 - \alpha_0)}{3\beta_0'^2B^2} - \frac{5}{2b^4\alpha_0'^2}\right],$$

$$c_9 = \frac{4\mu\Phi}{a^3B} + \frac{2\mu\Phi^2}{3a^4b^4\beta_0'B^2}\left[4b^2(a^2 + b^2)\beta_0' - 15(a^2 - b^2)\alpha_0\right],$$

$$c_{10} = -\frac{4\mu\Phi}{ab^2B} - \frac{2\mu\Phi^2}{3a^4b^4\beta_0'B^2}\left[4a^2(a^2 + b^2)\beta_0' + 15(a^2 - b^2)\beta_0\right],$$

$$\lambda_1 = \frac{-2ab^2\beta_0''G\,\dfrac{a}{\alpha_0}\left(1 - \dfrac{q_0}{q}\right)}{\mu\left(2 + 3ab^2\beta_0''\dfrac{\eta}{\mu}\right)}(1 - M\Phi), \quad \lambda_2 = \frac{2 + (N - 2M)\Phi}{a^2\left(2 + 3ab^2\beta_0''\dfrac{\eta}{\mu}\right)} -$$

$$- \frac{a^2 - b^2 + \dfrac{15(a^2 - b^2)(\alpha_0 + \beta_0) + 4(a^2 + b^2)(\beta_0 - \alpha_0)}{6ab^2\beta_0'B}\Phi}{a^2 + b^2 + \dfrac{15(a^2 - b^2)^2 + 4(a^2 + b^2)^2}{6ab^2B}\Phi},$$

$$\lambda_3 = \frac{a^2 - b^2 + \dfrac{15(a^2 - b^2)(\alpha_0 + \beta_0) + 4(a^2 + b^2)(\beta_0 - \alpha_0)}{6ab^2\beta_0'B}\Phi}{a^2 + b^2 + \dfrac{15(a^2 - b^2)^2 + 4(a^2 + b^2)^2}{6ab^2B}\Phi},$$

$$\lambda_4 = \frac{3B}{16\pi\mu\left[a^2 + b^2 + \dfrac{15(a^2 - b^2)^2 + 4(a^2 + b^2)}{6ab^2B}\Phi\right]},$$

$$\lambda_5 = \frac{3a\beta_0''}{4\pi\mu\left(2 + 3ab^2\beta_0''\dfrac{\eta}{\mu}\right)}(1 - M\Phi).$$

Since the vector of the orientation n_i characterizes the behavior of the microstructure, to obtain the rheological equations of state, averaging must be carried out in (8) using the distribution function of the angular positions and the lengths of the axis of rotation of the suspended particles. Here, the stress tensor of the suspensions under consideration assumes the form

$$T_{ij} = \langle t_{ij}\rangle = [c_0 + \langle c_1n_kn_m\rangle d_{km} + \langle c_2N_kn_k\rangle]\delta_{ij} + \langle c_4n_kn_mn_in_j\rangle d_{km} +$$
$$+ \langle c_5N_kn_kn_in_j\rangle + \langle c_6\rangle d_{ij} + \langle c_7n_kn_j\rangle d_{ik} + \langle c_8n_kn_i\rangle d_{jk} + \langle c_9n_iN_j\rangle + \langle c_{10}n_jN_i\rangle, \tag{10}$$

where < > is the sign of averaging, using the distribution function of the angular positions and lengths of the axis of symmetry of the suspended particle F, in the case under consideration satisfying the equation [1]

$$\frac{\partial F}{\partial t} + kT(\lambda_4 - \lambda_5)n_in_j\frac{\partial^2F}{\partial n_i\partial n_j} + kT\lambda_4n^2\frac{\partial^2F}{\partial n_i\partial n_j} + \lambda_2n_i\frac{\partial F}{\partial n_i}d_{km}n_kn_m + \left[\lambda_1 + kT\left(2\lambda_4 - 4\lambda_5 - n\frac{d\lambda_5}{dn}\right)\right]n_i\frac{\partial F}{\partial n_i} +$$
$$+ (\omega_{ij} + \lambda_3d_{ij})n_j\frac{\partial F}{\partial n_i} + \left(3\lambda_1 + n\frac{d\lambda_1}{dn}\right)F + \left(5\lambda_2 + n\frac{d\lambda_2}{dn} + \frac{1}{n}\frac{d\lambda_3}{dn}\right)Fd_{km}n_kn_m = 0, \tag{11}$$

where t is the time.

In conclusion we note that, taking account of the hydrodynamic interaction of suspended particles by the method proposed in the present article, the rheological equations of state of weakly concentrated suspensions of deformable ellipsoidal particles (10) and the equation for the distribution function coincide in form with the corresponding equations for dilute suspensions of such particles. The hydrodynamic interaction of suspended particles manifests itself in a change in the rheological functions entering into these equations.

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INVESTIGATION OF THE DECOMPOSITION OF JETS OF RHEOLOGICALLY COMPLEX LIQUIDS

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1. Experimental Investigation of the Decomposition of Jets of Pseudoplastic Liquids

The investigated liquids were suspensions with different concentrations of acicular iron gamma-oxide (γ-Fe_2O_3) in AMG-10 hydraulic fluid.

The rheological characteristics of the investigated materials are given in Fig. 1. Curves 1-3 show the dependence of the effective viscosity η on the shear rate $\dot{\gamma}$, respectively, for 18, 25, 36% suspensions by weight of γ-Fe_2O_3. This same dependence for a suspension of clay is shown by curve 4. The curves of the flow have a form characteristic for the rheograms of typical pseudoplastic media with a strong dependence of the viscosity on the shear rate. With a sufficient degree of exactness, this dependence can be described by a power function

$$\tau = K\dot{\gamma}^n,\ \eta = K\dot{\gamma}^{n-1}. \tag{1.1}$$

In an experiment, the carefully degassed liquid from a tank, under the action of a piston, set into motion by compressed air, is fed vertically downward through a nozzle with a diameter of 1.28 mm. The rate of outflow of the jet formed was high enough so that the acceleration due to gravity could be neglected, and small enough so that, with the limits of the recorded section, there arose no significant aerodynamic perturbations. At the outlet from the nozzle, using a thin needle, which, at a right angle, touches the surface of the jet, and which is brought into motion by an electrodynamic vibrator, periodic perturbations, controlled in amplitude and frequency, are applied to the jet. For clay suspensions, perturbations were set up with excitation at the resonance frequency of a vibrator attached on the frame of the unit. The fully established periodic process of the decomposition of jets was recorded by photography of the jet with pulsed illumination against the background of a screen and a linear scale. The exposure time was 1 μsec.

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